Introduction to ROBOTICS

Manipulator Dynamics

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Outline

- Homework Highlight
- Review
  - Kinematics Model
  - Jacobian Matrix
  - Trajectory Planning
- Dynamic Model
  - Langrange-Euler Equation
  - Examples

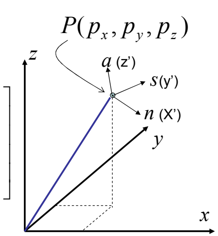
Homework highlight

- Composite Homogeneous Transformation Matrix Rules:
  - Transformation (rotation/translation) w.r.t. (X,Y,Z) (OLD FRAME), using **pre-multiplication**
  - Transformation (rotation/translation) w.r.t. (U,V,W) (NEW FRAME), using **post-multiplication**

Homework Highlight

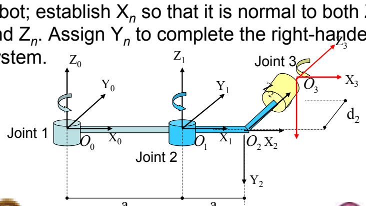
- Homogeneous Representation
  - A frame in R^3 space

$$F = \begin{bmatrix} n & s & a & P \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Homework Highlight

- Assign $Y_i = +(Z_i \times X_i)/\|Z_i \times X_i\|$ to complete the right-handed coordinate system.
- The hand coordinate frame is specified by the geometry of tool. Normally, establish Z_n along the direction of Z_{n-1} axis and pointing away from the robot; establish X_n so that it is normal to both Z_{n-1} and Z_n. Assign Y_n to complete the right-handed system. O_n



Review

- Steps to derive kinematics model:
  - Assign D-H coordinates frames
  - Find link parameters
  - Transformation matrices of adjacent joints
  - Calculate kinematics matrix
  - When necessary, Euler angle representation

Review

- D-H transformation matrix for adjacent coordinate frames, *i* and *i-1*.
  - The position and orientation of the *i*-th frame coordinate can be expressed in the *(i-1)*th frame by the following 4 successive elementary transformations:

$$T_{i-1}^{i} = T(z_{i-1}, d_i)R(z_{i-1}, \theta_i)T(x_i, a_i)R(x_i, \alpha_i)$$

$$= \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Review

- Kinematics Equations
  - chain product of successive coordinate transformation matrices of T_{i-1}^{i}
  - T_0^n specifies the location of the *n*-th coordinate frame w.r.t. the base coordinate system

$$T_0^n = T_0^1 T_1^2 \ldots T_{n-1}^n$$

$$= \begin{bmatrix} R_0^n & P_0^n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} n & s & a & P_0^n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orientation matrix

Position vector

Jacobian Matrix

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \xrightarrow{\text{Forward}} \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \varphi \end{bmatrix}$$

Kinematics

Inverse

Joint Space — Task Space

$$\begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot\theta_6 \end{bmatrix} \overset{\text{Jacobian Matrix}}{\longleftrightarrow} \begin{bmatrix} \dot x \\ \dot y \\ \dot z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Jaconian Matrix: Relationship between joint space velocity with task space velocity

Jacobian Matrix

$$\begin{bmatrix} \dot x \\ \dot y \\ \dot z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \left[\frac{dh(q)}{dq}\right]_{6\times n} \begin{bmatrix} \dot q_1 \\ \dot q_2 \\ \vdots \\ \dot q_n \end{bmatrix}_{n\times 1}$$

Jacobian is a function of q, it is not a constant!

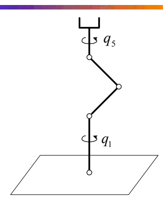
$$J = \left(\frac{dh(q)}{dq}\right)_{6\times n} = \begin{bmatrix} \frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \cdots & \frac{\partial h_1}{\partial q_n} \\ \frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \cdots & \frac{\partial h_2}{\partial q_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_6}{\partial q_1} & \frac{\partial h_6}{\partial q_2} & \cdots & \frac{\partial h_6}{\partial q_n} \end{bmatrix}_{6\times n}$$

Jacobian Matrix

- Inverse Jacobian

$$\dot Y = J\dot q = \begin{bmatrix} J_{11} & J_{12} & \cdots & J_{16} \\ J_{21} & J_{22} & \cdots & J_{26} \\ \vdots & \vdots & \vdots & \vdots \\ J_{61} & J_{62} & \cdots & J_{66} \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \dot q_2 \\ \dot q_3 \\ \dot q_4 \\ \dot q_5 \\ \dot q_6 \end{bmatrix}$$

$$\dot q = J^{-1}\dot Y$$



- Singularity
  - rank(J)<min{6,n},
  - Jacobian Matrix is less than full rank
  - Jacobian is non-invertable
  - Occurs when two or more of the axes of the robot form a straight line, i.e., collinear
  - Avoid it

Trajectory Planning

- Trajectory planning,
  - “interpolate” or “approximate” the desired path by a class of polynomial functions and generates a sequence of time-based “control set points” for the control of manipulator from the initial configuration to its destination.
  - Requirements: Smoothness, continuity
  - Piece-wise polynomial interpolate
  - 4-3-4 trajectory

$$h_1(t) = a_{14}t^4 + a_{13}t^3 + a_{12}t^2 + a_{11}t + a_{10}$$
$$h_2(t) = a_{23}t^3 + a_{22}t^2 + a_{21}t + a_{20}$$
$$h_3(t) = a_{34}t^4 + a_{33}t^3 + a_{32}t^2 + a_{31}t + a_{30}$$

Manipulator Dynamics

- Mathematical equations describing the dynamic behavior of the manipulator
  - For computer simulation
  - Design of suitable controller
  - Evaluation of robot structure
  - Joint torques ⟺ Robot motion, i.e. acceleration, velocity, position

Manipulator Dynamics

- Lagrange-Euler Formulation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i$$

  - Lagrange function is defined

$$L = K - P$$

    - *K*: Total kinetic energy of robot
    - *P*: Total potential energy of robot
    - q_i: Joint variable of i-th joint
    - $\dot{q}_i$: first time derivative of q_i
    - τ_i : Generalized force (torque) at i-th joint

Manipulator Dynamics

- Kinetic energy
  - Single particle: $k = \frac{1}{2} m v^2$
  - Rigid body in 3-D space with linear velocity (V) , and angular velocity (ω) about the center of mass

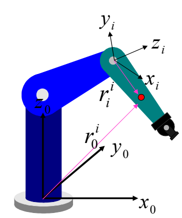
$$k = \frac{1}{2} m V^T V + \frac{1}{2} I \omega^T \omega$$

$$I \equiv \begin{bmatrix} \int (y^2+z^2)dm & -\int xydm & -\int xzdm \\ -\int xydm & \int (x^2+z^2)dm & -\int yzdm \\ -\int xzdm & -\int yzdm & \int (x^2+y^2)dm \end{bmatrix}$$

  - ***I*** : Inertia Tensor: $I_{xx} = \int (y^2+z^2)dm$
    - Diagonal terms: moments of inertia
    - Off-diagonal terms: products of inertia $I_{xy} = \int (xy)dm$

Velocity of a link

$$r_i^i = \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$ A point fixed in link *i* and expressed w.r.t. the *i*-th frame



Same point w.r.t the base frame

$$r_0^i = T_0^i r_i^i = (T_0^1 T_1^2 \cdots T_{i-1}^i) r_i^i$$

Velocity of a link

Velocity of point r_i^i expressed w.r.t. *i*-th frame is zero

$$\dot{r}_i^i = 0$$

Velocity of point r_i^i expressed w.r.t. base frame is:

$$V_i \equiv V_0^i = \frac{d}{dt} r_0^i = \frac{d}{dt}(T_0^1 T_1^2 \cdots T_{i-1}^i) r_i^i$$

$$= \dot{T}_0^1 T_1^2 \cdots T_{i-1}^i r_i^i + T_0^1 \dot{T}_1^2 \cdots T_{i-1}^i r_i^i +$$

$$\cdots + T_0^1 T_1^2 \cdots \dot{T}_{i-1}^i r_i^i + T_0^i \dot{r}_i^i = \left(\sum_{j=1}^{i} \frac{\partial T_0^i}{\partial q_j} \dot{q}_j\right) r_i^i$$

Velocity of a link

- Rotary joints, $q_i = \theta_i$

$$T_{i-1}^i = \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⇓

$$\frac{\partial T_{i-1}^i}{\partial q_i} = \begin{bmatrix} -S\theta_i & -C\alpha_i C\theta_i & S\alpha_i C\theta_i & -a_i S\theta_i \\ C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_i = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial T_{i-1}^i}{\partial q_i} = Q_i T_{i-1}^i = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Velocity of a link

- Prismatic joint, $q_i = d_i$

$$T_{i-1}^{i} = \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial T_{i-1}^{i}}{\partial q_i} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \frac{\partial T_{i-1}^{i}}{\partial q_i} = Q_i T_{i-1}^{i}$$

Velocity of a link

The effect of the motion of joint j on all the points on link i

$$\frac{\partial T_0^i}{\partial q_j} = \begin{cases} T_0^1 T_1^2 \cdots T_{j-2}^{j-1} Q_j T_{j-1}^{j} \cdots T_{i-1}^{i} & for \quad j \le i \\ 0 & for \quad j > i \end{cases}$$

$$U_{ij} \equiv \frac{\partial T_0^i}{\partial q_j} = \begin{cases} T_0^{j-1} Q_j T_{j-1}^{i} & for \quad j \le i \\ 0 & for \quad j > i \end{cases}$$

$$V_i \equiv V_0^i = \frac{d}{dt} r_0^i = \frac{d}{dt}(T_0^1 T_1^2 \cdots T_{i-1}^{i}) r_i^i = (\sum_{j=1}^{i} \frac{\partial T_0^i}{\partial q_j} \dot{q}_j) r_i^i = (\sum_{j=1}^{i} U_{ij} \dot{q}_j) r_i^i$$

Kinetic energy of link i

- Kinetic energy of a particle with differential mass *dm* in link i

$$dK_i = \frac{1}{2}(\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) dm = \frac{1}{2} trace(V_i V_i^T) dm$$

$$= \frac{1}{2} Tr \left[\sum_{p=1}^{i} U_{ip} \dot{q}_p r_i^i (\sum_{r=1}^{i} U_{ir} \dot{q}_r r_i^i)^T \right] dm$$

$$= \frac{1}{2} Tr \left[\sum_{p=1}^{i} \sum_{r=1}^{i} U_{ip} r_i^i r_i^{iT} U_{ir}^T \dot{q}_p \dot{q}_r \right] dm$$

$$= \frac{1}{2} Tr \left[\sum_{p=1}^{i} \sum_{r=1}^{i} U_{ip} (r_i^i dm r_i^{iT}) U_{ir}^T \dot{q}_p \dot{q}_r \right]$$

$$Tr(A) \equiv \sum_{i=1}^{n} a_{ii}$$

Kinetic energy of link i

$$K_i = \int dK_i = \frac{1}{2} Tr \left[\sum_{p=1}^{i} \sum_{r=1}^{i} U_{ip} (\int r_i^i r_i^{iT} dm) U_{ir}^T \dot{q}_p \dot{q}_r \right]$$

$$J_i = \int r_i^i r_i^{iT} dm = \begin{bmatrix} \int x_i^2 dm & \int x_i y_i dm & \int x_i z_i dm & \int x_i dm \\ \int x_i y_i dm & \int y_i^2 dm & \int y_i z_i dm & \int y_i dm \\ \int x_i z_i dm & \int y_i z_i dm & \int z_i^2 dm & \int z_i dm \\ \int x_i dm & \int y_i dm & \int z_i dm & \int dm \end{bmatrix} \qquad \bar{r}_i^i = \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \\ \bar{z}_i \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-I_{xx} + I_{yy} + I_{zz}}{2} & I_{xy} & I_{xz} & m_i \bar{x}_i \\ I_{xy} & \frac{I_{xx} - I_{yy} + I_{zz}}{2} & I_{yz} & m_i \bar{y}_i \\ I_{xz} & I_{yz} & \frac{I_{xx} + I_{yy} - I_{zz}}{2} & m_i \bar{z}_i \\ m_i \bar{x}_i & m_i \bar{y}_i & m_i \bar{z}_i & m_i \end{bmatrix}$$

$$\bar{x}_i = \frac{1}{m_i} \int x_i dm$$

Center of mass

Pseudo-inertia matrix of link i

Manipulator Dynamics

- Total kinetic energy of a robot arm

$$K = \sum_{i=1}^{n} K_i = \frac{1}{2} \sum_{i=1}^{n} Tr \left[\sum_{p=1}^{i} \sum_{r=1}^{i} U_{ip} (\int r_i^i r_i^{iT} dm) U_{ir}^T \dot{q}_p \dot{q}_r \right]$$

$$= \frac{1}{2} \sum_{i=1}^{n} \sum_{p=1}^{i} \sum_{r=1}^{i} \left[Tr(U_{ip} J_i U_{ir}^{\ T}) \dot{q}_p \dot{q}_r \right]$$

Scalar quantity, function of q_i and $\dot{q}_i$, $i = 1, 2, \cdots n$

J_i : Pseudo-inertia matrix of link i, dependent on the mass distribution of link i and are expressed w.r.t. the i-th frame,

Need to be computed once for evaluating the kinetic energy

Manipulator Dynamics

- Potential energy of link i

$$P_i = -m_i g \bar{r}_0^i = -m_i g (T_0^i \bar{r}_i^i)$$

$\bar{r}_0^i$: Center of mass w.r.t. base frame

$\bar{r}_i^i$: Center of mass w.r.t. i-th frame

$$g = (g_x, g_y, g_z, 0)$$

$$|g| = 9.8 m/\sec^2$$

g : gravity row vector expressed in base frame

- Potential energy of a robot arm

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} [-m_i g (T_0^i \bar{r}_i^i)]$$

Function of q_i

Manipulator Dynamics

- Lagrangian function

$$L = K - P = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i}\left[Tr(U_{ij}J_iU_{ik}^T)\dot{q}_j\dot{q}_k\right] + \sum_{i=1}^{n} m_i g(T_0^i \bar{r}_i^i)$$

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

$$= \sum_{j=i}^{n}\sum_{k=1}^{j} Tr(U_{jk}J_jU_{ji}^T)\ddot{q}_k + \sum_{j=i}^{n}\sum_{k=1}^{j}\sum_{m=1}^{j} Tr\left(\frac{\partial U_{jk}}{\partial q_m}J_jU_{ji}^T\right)\dot{q}_k\dot{q}_m$$

$$-\sum_{j=i}^{n} m_j g U_{ji}\bar{r}_j^j$$

Manipulator Dynamics

The effect of the motion of joint j on all the points on link i

$$U_{ij} \equiv \frac{\partial T_0^i}{\partial q_j} = \begin{cases} T_0^{j-1}Q_jT_{j-1}^i & for \quad j \le i \\ 0 & for \quad j > i \end{cases}$$

The interaction effects of the motion of joint j and joint k on all the points on link i

$$\frac{\partial U_{ij}}{\partial q_k} \equiv U_{ijk} = \begin{cases} T_0^{j-1}Q_jT_{j-1}^{k-1}Q_kT_{k-1}^i & i \ge k \ge j \\ T_0^{k-1}Q_kT_{k-1}^{j-1}Q_jT_{j-1}^i & i \ge j \ge k \\ 0 \qquad i < j \quad or \quad i < k & \end{cases}$$

Manipulator Dynamics

- Dynamics Model

$$\tau_i = \sum_{k=1}^{n} D_{ik}\ddot{q}_k + \sum_{k=1}^{n}\sum_{m=1}^{n} h_{ikm}\dot{q}_k\dot{q}_m + C_i$$

$$D_{ik} = \sum_{j=\max(i,k)}^{n} Tr(U_{jk}J_jU_{ji}^T)$$

$$h_{ikm} = \sum_{j=\max(i,k,m)}^{n} Tr(U_{jkm}J_jU_{ji}^T)$$

$$C_i = -\sum_{j=i}^{n} m_j g U_{ji}\bar{r}_j^j$$

Manipulator Dynamics

- Dynamics Model of n-link Arm

$$\tau = D(q)\ddot{q} + h(q,\dot{q}) + C(q)$$

$$D = \begin{bmatrix} D_{11} & \cdots & D_{1n} \\ & \vdots & \\ D_{n1} & \cdots & D_{nn} \end{bmatrix}$$ The Acceleration-related Inertia matrix term, Symmetric

$$h(q,\dot{q}) = \begin{bmatrix} h_1 \\ \vdots \\ h_n \end{bmatrix}$$ The Coriolis and Centrifugal terms

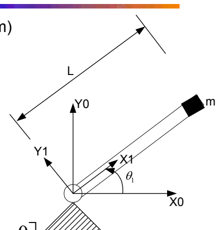
$$C(q) = \begin{bmatrix} C_1 \\ \vdots \\ C_n \end{bmatrix}$$ The Gravity terms

$$\tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_n \end{bmatrix}$$ Driving torque applied on each link

Example

Example: One joint arm with point mass (m) concentrated at the end of the arm, link length is l , find the dynamic model of the robot using L-E method.

Set up coordinate frame as in the figure

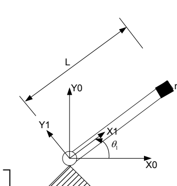


$$r_1^1 = \begin{bmatrix} l \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad g = [0, -9.8, 0, 0]$$

$$r_0^1 = T_0^1 r_1^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} r_1^1$$

Example

$$r_0^1 = T_0^1 r_1^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} r_1^1$$



$$V_1 = \dot{r}_0^1 = \frac{d}{dt}T_0^1 r_1^1 = Q_1 T_0^1 r_1^1 = \begin{bmatrix} -l \cdot S\theta_1 \\ l \cdot C\theta_1 \\ 0 \\ 0 \end{bmatrix} \dot{\theta}$$

Example

Kinetic energy

$$dK = \frac{1}{2} Tr(V_1 V_1^T) dm$$

$$K = \frac{1}{2}\int Tr(\begin{bmatrix} -l\cdot S\theta_1 \\ l\cdot C\theta_1 \\ 0 \\ 0 \end{bmatrix}[-l\cdot S\theta_1 \quad l\cdot C\theta_1 \quad 0 \quad 0])\dot{\theta}_1^2 dm$$

$$= \frac{1}{2} Tr(\begin{bmatrix} l^2\cdot(S\theta_1)^2 & -l^2\cdot S\theta_1\cdot C\theta_1 & 0 & 0 \\ -l^2\cdot S\theta_1\cdot C\theta_1 & l^2\cdot(C\theta_1)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix})\dot{\theta}_1^2 m$$

$$= \frac{1}{2}[l^2(S\theta_1)^2 + l^2(C\theta_1)^2] m\dot{\theta}^2 = \frac{1}{2} l^2 m \dot{\theta}_1^2$$

Example

- Potential energy

$$P = -mg(T_0^1 \bar{r}_1) = -m[0 \quad -9.8 \quad 0 \quad 0]\begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} l \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= 9.8m\cdot l\cdot S\theta_1$$

- Lagrange function

$$L = K - P = \frac{1}{2} l^2 m\dot{\theta}_1^2 - 9.8m\cdot l\cdot S\theta_1$$

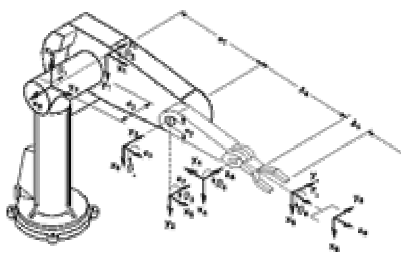
- Equation of Motion

$$\tau_1 = \frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_1}) - \frac{\partial L}{\partial \theta_1}$$

$$= \frac{d}{dt}(l^2 m\dot{\theta}_1) - 9.8m\cdot l\cdot C\theta_1 = l^2 m\ddot{\theta}_1 - 9.8m\cdot l\cdot C\theta_1$$

Example: **Puma 560**

- Derive dynamic equations for the first 4 links of PUMA 560 robot



Example: **Puma 560**

- Set up D-H Coordinate frame
- Get robot link parameters

Joint i	θ_i	α_i	a_i*(mm)*	d_i*(mm)*
1	θ_1	-90	0	0
2	θ_2	0	431.8	-149.09
3	θ_3	90	-20.32	0
4	θ_4	-90	0	433.07
5	θ_5	90	0	0
6	θ_6	0	0	56.25

- Get transformation matrices T_{i-1}^i
- Get D, H, C terms

Example: **Puma 560**

- Get D, H, C terms

$$D_{ik} = \sum_{j=\max(i,k)}^{n} Tr(U_{jk} J_j U_{ji}^T) \qquad n = 3; i = 1,2,3$$

$$D_{11} = Tr(U_{11}J_1U_{11}^T) + Tr(U_{21}J_2U_{21}^T) + Tr(U_{31}J_3U_{31}^T)$$

$$D_{12} = D_{21} = Tr(U_{22}J_2U_{21}^T) + Tr(U_{32}J_3U_{31}^T)$$

$$D_{13} = D_{31} = Tr(U_{33}J_3U_{31}^T)$$

$$D_{22} = Tr(U_{22}J_2U_{22}^T) + Tr(U_{32}J_3U_{32}^T)$$

$$D_{23} = D_{32} = Tr(U_{33}J_3U_{32}^T)$$

$$D_{33} = Tr(U_{33}J_3U_{33}^T)$$

Example: **Puma 560**

- Get D, H, C terms

$$h(q,\dot{q}) = \begin{bmatrix} h_1 \\ \vdots \\ h_n \end{bmatrix} \qquad h_i = \sum_{k=1}^{n}\sum_{m=1}^{n} h_{ikm}\dot{q}_k\dot{q}_m \qquad h_{ikm} = \sum_{j=\max(i,k,m)}^{n} Tr(U_{jkm}J_jU_{ji}^T)$$

$$h_1 = h_{111}\dot{q}_1^2 + h_{112}\dot{q}_1\dot{q}_2 + h_{113}\dot{q}_1\dot{q}_3 + h_{121}\dot{q}_1\dot{q}_2 + h_{122}\dot{q}_2^2$$
$$+ h_{123}\dot{q}_2\dot{q}_3 + h_{131}\dot{q}_3\dot{q}_1 + h_{132}\dot{q}_3\dot{q}_2 + h_{133}\dot{q}_3^2$$

$$h_{111} = Tr(U_{111}J_1U_{11}^T) + Tr(U_{211}J_2U_{21}^T) + Tr(U_{311}J_3U_{31}^T)$$

$$h_{112} = Tr(U_{212}J_2U_{21}^T) + Tr(U_{312}J_3U_{31}^T)$$

$$h_{121} = Tr(U_{221}J_2U_{21}^T) + Tr(U_{321}J_3U_{31}^T) \qquad h_{113} = Tr(U_{313}J_3U_{31}^T)$$

$$h_{122} = Tr(U_{222}J_2U_{21}^T) + Tr(U_{322}J_3U_{31}^T) \qquad h_{123} = Tr(U_{323}J_3U_{31}^T)$$

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Example: **Puma 560**

- Get D, H, C terms

$$\frac{\partial U_{ij}}{\partial q_k} \equiv U_{ijk} = \begin{cases} T_0^{j-1} Q_j T_{j-1}^{k-1} Q_k T_{k-1}^{i} & i \ge k \ge j \\ T_0^{k-1} Q_k T_{j-1}^{k-1} Q_j T_{j-1}^{i} & i \ge j \ge k \\ 0 & i < j \quad or \quad i < k \end{cases}$$

$$U_{111} = (Q_1)^2 T_0^1 \qquad U_{211} = (Q_1)^2 T_0^2 \qquad U_{311} = (Q_1)^2 T_0^3$$

$$U_{212} = U_{221} = Q_1 T_0^1 Q_2 T_1^2 \qquad U_{312} = U_{321} = Q_1 T_0^1 Q_2 T_1^3$$

$$U_{313} = Q_1 T_0^2 Q_3 T_2^3 \qquad U_{222} = T_0^1 (Q_2)^2 T_1^2 \qquad U_{322} = T_0^1 (Q_2)^2 T_1^3$$

$$U_{323} = U_{332} = T_0^1 Q_2 T_1^2 Q_3 T_2^3 \qquad U_{331} = Q_1 T_0^2 Q_3 T_2^3 \qquad U_{333} = T_0^2 Q_3 Q_2 T_2^3$$

Example: **Puma 560**

- Get D, H, C terms

$$C_i = -\sum_{j=i}^{n} m_j g U_{ji} \bar{r}_j^j$$

$$C_1 = -m_1 g U_{11} \bar{r}_1^1 - m_2 g U_{21} \bar{r}_2^2 - m_3 g U_{31} \bar{r}_3^3$$

$$C_2 = -m_2 g U_{22} \bar{r}_2^2 - m_3 g U_{32} \bar{r}_3^3$$

$$C_3 = -m_3 g U_{33} \bar{r}_3^3$$

Thank you!

Homework 4 posted on the web.

Due: Oct. 7, 2005 (Friday)

Next class: Manipulator Control