Lecture 2: Digital Image Fundamentals -- Sampling & Quantization

Prof. YingLi Tian
Sept. 6, 2017

Some materials from Dr. Lexing Xie and Dr. Shahram Ebadollahi
Announcement

- Chapter 2 (please read) – this week
- Course website: http://www-ee.ccny.cuny.edu/www/web/yltian/I2200.html
- Book website: http://www.prenhall.com/gonzalezwoods
- HW1 is out today:
  - Due: 9/26/2017
Outline:

- Illusions
- Image sampling and interpolation
- Image quantization
- Basic relationship between Pixels
- Basic operations for image processing
Cognitive illusions of the human visual system
Cognitive illusions of the human visual system
Cognitive illusions of the human visual system
Cognitive illusions of the human visual system

**FIGURE 2.8** Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.
Cognitive illusions of the human visual system
Confused color

red  blue  orange  purple
orange  blue  green  red
blue  purple  green  red
orange  blue  red  green
purple  orange  red  blue
purple  orange  red  blue
green  red  blue  purple
green  red  blue  purple
orange  blue  red  green
green  purple  orange  red

Read out the color of the words, not the word itself.

How far can you go?
Digital Image Acquisition: From Physical Image to Digital Image

**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy ("illumination") source, (b) An element of a scene, (c) Imaging system, (d) Projection of the scene onto the image plane, (e) Digitized image.
Digital Image Acquisition: From Physical Image to Digital Image

- Film
- Image sensors
  - CCD (Charge-coupled device)
  - CMOS (Complementary Metal–Oxide–Semiconductor)
    - http://en.wikipedia.org/wiki/Active_pixel_sensor
  - Color Cameras: Bayer color filter
Digital Image Acquisition: CCD Sensor

Single imaging sensor

Line sensor

Array sensor
CCD vs CMOS

- A CCD is an analog device. When light strikes the chip it is held as a small electrical charge in each photo sensor. The charges are converted to voltage one pixel at a time as they are read from the chip. Additional circuitry in the camera converts the voltage into digital information.

- A CMOS chip is a type of active pixel sensor made using the CMOS semiconductor process. Extra circuitry next to each photo sensor converts the light energy to a voltage. Additional circuitry on the chip may be included to convert the voltage to digital data.
Bayer color filter

- Invented by B. E. Bayer at Kodak,
  For more details:
  http://www.siliconimaging.com/RGB%20Bayer.htm
Digital Image Acquisition: From Physical Image to Digital Image

Goal: generate digital images from sensed data

- **Sampling**
  
  Process of mapping a continuous function to discrete – coordinate values

  NOTE: pixels are samples from physical image

  - Photoreceptors in eye
  - CCD array

- **Quantization**
  
  Process of mapping continuous variable to discrete – digitizing the amplitude values
Digital Image Acquisition: From Physical Image to Digital Image

http://www.cs.princeton.edu/courses/archive/fall00/cs426/lectures/dither/dither.pdf
Image Resolution

- Intensity resolution
  - Each pixel has only “Depth” bits for colors/intensities

- Spatial resolution
  - Image has only “Width” x “Height” pixels

- Temporal resolution
  - Monitor refreshes images at only “Rate” Hz

<table>
<thead>
<tr>
<th>Typical Resolutions</th>
<th>Width x Height</th>
<th>Depth</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTSC</td>
<td>640 x 480</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>Workstation</td>
<td>1280 x 1024</td>
<td>24</td>
<td>75</td>
</tr>
<tr>
<td>Film</td>
<td>3000 x 2000</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Laser Printer</td>
<td>6600 x 5100</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>
Digital Image Acquisition: From Physical Image to Digital Image

Spatial discretization

Intensity discretization
Digital Image Acquisition: From Physical Image to Digital Image

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.
FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).
Image representation - Array

\[ f = \begin{bmatrix}
  f(0,0) & f(0,1) & \cdots & f(0,N-1) \\
  f(1,0) & f(1,1) & \cdots & f(1,N-1) \\
  \vdots & \vdots & \ddots & \vdots \\
  f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1)
\end{bmatrix} \]

\[ 0 \leq f \leq L-1 \]
\[ 0 \leq x \leq M-1 \quad 0 \leq y \leq N-1 \]
Image Spatial and Intensity Resolution

Spatial resolution:

\[ 0 \leq x \leq M-1 \]
\[ 0 \leq y \leq N-1 \]

Note: change spatial resolution is called **sampling**.

Intensity resolution:

\[ 0 \leq f \leq L-1 \quad \text{L is the number of intensity levels} \]

\[ L = 2^k \]

Note: change intensity resolution is called **quantization**.
Image Storage

The number, \( b \), of bits required to store a digitized \( M \) by \( N \) image

\[
b = M \times N \times k
\]

When \( M = N \),

\[
b = N^2 k
\]

**Question:** 1 byte = ? bits

1 byte = 8 bits
# Image Storage

## Table 2.1

Number of storage bits for various values of $N$ and $k$.

| $N/k$ | 1 ($L = 2$) | 2 ($L = 4$) | 3 ($L = 8$) | 4 ($L = 16$) | 5 ($L = 32$) | 6 ($L = 64$) | 7 ($L = 128$) | 8 ($L = 256$) |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 32    | 1,024       | 2,048       | 3,072       | 4,096       | 5,120       | 6,144       | 7,168       | 8,192       |
| 64    | 4,096       | 8,192       | 12,288      | 16,384      | 20,480      | 24,576      | 28,672      | 32,768      |
| 128   | 16,384      | 32,768      | 49,152      | 65,536      | 81,920      | 98,304      | 114,688     | 131,072     |
| 256   | 65,536      | 131,072     | 196,608     | 262,144     | 327,680     | 393,216     | 458,752     | 524,288     |
| 512   | 262,144     | 524,288     | 786,432     | 1,048,576   | 1,310,720   | 1,572,864   | 1,835,008   | 2,097,152   |
| 1024  | 1,048,576   | 2,097,152   | 3,145,728   | 4,194,304   | 5,242,880   | 6,291,456   | 7,340,032   | 8,388,608   |
| 2048  | 4,194,304   | 8,388,608   | 12,582,912  | 16,777,216  | 20,971,520  | 25,165,824  | 29,369,128  | 33,554,432  |
| 4096  | 16,777,216  | 33,554,432  | 50,331,648  | 67,108,864  | 83,886,080  | 100,663,296 | 117,440,512 | 134,217,728 |
| 8192  | 67,108,864  | 134,217,728 | 201,326,592 | 268,435,456 | 335,544,320 | 402,653,184 | 469,762,048 | 536,870,912 |
Neighbors of a pixel

<table>
<thead>
<tr>
<th></th>
<th>(x-1, y-1)</th>
<th>(x-1, y)</th>
<th>(x-1, y+1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x, y-1)</td>
<td>(x, y)</td>
<td>(x, y+1)</td>
<td></td>
</tr>
<tr>
<td>(x+1, y-1)</td>
<td>(x+1, y)</td>
<td>(x+1, y+1)</td>
<td></td>
</tr>
</tbody>
</table>
Examples of Image of Different Spatial Resolution
Examples of Image of Different Intensity Resolution

1-bit

2-bit

3-bit

4-bit

5-bit

6-bit

7-bit

8-bit
Image Spatial Sampling -- resizing

- High resolution $\rightarrow$ low resolution
  - Down-sample or shrink
- Low resolution $\rightarrow$ high resolution
  - Interpolation or zoom in
Image Interpolation

Interpolation works by using known data to estimate values at unknown points.

- Linear interpolation

For example: if you wanted to know the temperature at noon, but only measured it at 11AM and 1PM,
Image Interpolation

If you had an additional measurement at 11:30AM, you could see that the bulk of the temperature rise occurred before noon, and could use this additional data point to perform a quadratic interpolation:
Image Interpolation

Image interpolation works in two directions, and tries to achieve a best approximation of a pixel's color and intensity based on the values at surrounding pixels.

2D Interpolation

Original Before After No Interpolation

http://www.cambridgeincolour.com/tutorials/image-interpolation.htm
Image Resizing methods

- **Non-adaptive algorithms**
  - Nearest Neighbor Interpolation
  - Bilinear interpolation
  - Bicubic interpolation

- **Adaptive algorithms**
  - Commercial software
Nearest Neighbor Interpolation

- Nearest neighbor is the most basic and requires the least processing time of all the interpolation algorithms because it only considers one pixel-- the closest one to the interpolated point. This has the effect of simply making each pixel bigger.
Take 10 mins rest, then we’ll continue
Bilinear Interpolation

- Bilinear interpolation considers the closest 2x2 neighborhood of known pixel values surrounding the unknown pixel. It then takes a weighted average of these 4 pixels to arrive at its final interpolated value. This results in much smoother looking images than nearest neighbor.
Bicubic Interpolation

Bicubic goes one step beyond bilinear by considering the closest 4x4 neighborhood of known pixels-- for a total of 16 pixels. Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation.
**Bilinear Interpolation Method**

\[ f(0,0) = A \]
\[ f(1,0) = B \]
\[ f(0,1) = C \]
\[ f(1,1) = D \]

1. \[ f_1(p) = (1 - p)A + pB \]
2. \[ f_2(p) = (1 - p)C + pD \]

\[ f(p, q) = (1 - q)f_1(p) + qf_2(p) \]
\[ = (1 - p)(1 - q)A + p(1 - q)B + (1 - p)qC + pqD \]
Problem with Downsampling
Problem with Downsampling
Problem with Downsampling

Fold over frequencies

Aliasing Error

* Not correctable with post filtering
Aliasing Error – Visual Perception

Original image

Down-sampled version
OPTICAL vs. DIGITAL ZOOM

Many compact digital cameras can perform both an optical and a digital zoom. **Optical zoom** by moving the zoom lens so that it increases the magnification of light before it even reaches the digital sensor. **Digital zoom** degrades quality by simply interpolating the image-- after it has been acquired at the sensor.
OPTICAL vs. DIGITAL ZOOM

10X Optical Zoom

10X Digital Zoom
Quantization

- Artifacts due to limited intensity resolution
  - Frame buffers have limited number of bits per pixel
  - Physical devices have limited dynamic range
Uniform Quantization

\[ P(x, y) = \text{trunc}(I(x, y) + 0.5) \]
Uniform Quantization

Images with decreasing bits per pixel:

8 bits  4 bits  2 bits  1 bit

Reduce effects of quantization: Halftoning and Dithering
Classical Halftoning

- Use dots of varying size to represent intensities
  - Area of dots proportional to intensity in image
Classical Halftoning

Newspaper Image
Halftone patterns

- Use cluster of pixels to represent intensity
- Trade spatial resolution for intensity resolution
Halftone patterns

- How many intensities in a n x n cluster?

0
0 ≤ I < 0.1

1
0.1 ≤ I < 0.2

2
0.2 ≤ I < 0.3

3
0.3 ≤ I < 0.4

4
0.4 ≤ I < 0.5

5
0.5 ≤ I < 0.6

6
0.6 ≤ I < 0.7

7
0.7 ≤ I < 0.8

8
0.8 ≤ I < 0.9

9
0.9 ≤ I ≤ 1.0
Dithering

- Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities

Original (8 bits)  Uniform Quantization (1 bit)  Floyd-Steinberg Dither (1 bit)
Dithering: Random vs Ordered

Original (8 bits)

Random Dither (1 bit)

Ordered Dither (1 bit)
FIGURE 2.25 (a) An arrangement of pixels. (b) Pixels that are 8-adjacent (adjacency is shown by dashed lines; note the ambiguity). (c) m-adjacency. (d) Two regions that are adjacent if 8-adjacency is used. (e) The circled point is part of the boundary of the 1-valued pixels only if 8-adjacency between the region and background is used. (f) The inner boundary of the 1-valued region does not form a closed path, but its outer boundary does.
Edge

Original image  Sobel edge  Laplace edge
Boundary (border or contour)
Connected Component

- Connected components labeling scans an image and groups its pixels into components based on pixel connectivity.
Distance Measurement

Pixels $p(x, y)$, $q(s, t)$, and $z(v, w)$

- **Euclidean distance between $p$ and $q$**
  \[ D(p,q) = \sqrt{(x-s)^2 + (y-t)^2} \]

- **$D4$ distance**
  \[ D4(p,q) = |x - s| + |y - t| \]

- **$D8$ distance**
  \[ D8(p,q) = \max(|x - s|, |y - t|) \]
Image Difference

**FIGURE 2.27** (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.
Image difference and enhancement

FIGURE 2.28
Digital subtraction angiography.
(a) Mask image.
(b) A live image.
(c) Difference between (a) and (b).
(d) Enhanced difference image.
(Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)
Image Arithmetic

- Addition
- Subtraction
- Multiplication
- Division
- Logical: AND, OR, XOR, NOT,...
Background Subtraction

- Find moving objects, Video demo
Basic operations

**FIGURE 2.31**
(a) Two sets of coordinates, $A$ and $B$, in 2-D space. (b) The union of $A$ and $B$. (c) The intersection of $A$ and $B$. (d) The complement of $A$. (e) The difference between $A$ and $B$. In (b)–(e) the shaded areas represent the member of the set operation indicated.
Example of basic operations

**FIGURE 2.32** Set operations involving grayscale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)
Logical Operations

**FIGURE 2.33**
Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.
# Affine Transformation

**TABLE 2.2**
Affine transformations based on Eq. (2.6–23).

<table>
<thead>
<tr>
<th>Transformation Name</th>
<th>Affine Matrix, $T$</th>
<th>Coordinate Equations</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = v$ \quad $y = w$</td>
<td><img src="image" alt="Identity Example" /></td>
</tr>
<tr>
<td>Scaling</td>
<td>$\begin{bmatrix} c_x &amp; 0 &amp; 0 \ 0 &amp; c_y &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = c_x v$ \quad $y = c_y w$</td>
<td><img src="image" alt="Scaling Example" /></td>
</tr>
<tr>
<td>Rotation</td>
<td>$\begin{bmatrix} \cos \theta &amp; \sin \theta &amp; 0 \ -\sin \theta &amp; \cos \theta &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = v \cos \theta - w \sin \theta$ \quad $y = v \cos \theta + w \sin \theta$</td>
<td><img src="image" alt="Rotation Example" /></td>
</tr>
<tr>
<td>Translation</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ 0 &amp; 1 &amp; 0 \ t_x &amp; t_y &amp; 1 \end{bmatrix}$</td>
<td>$x = v + t_x$ \quad $y = w + t_y$</td>
<td><img src="image" alt="Translation Example" /></td>
</tr>
<tr>
<td>Shear (vertical)</td>
<td>$\begin{bmatrix} 1 &amp; 0 &amp; 0 \ s_y &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = v + s_y w$ \quad $y = w$</td>
<td><img src="image" alt="Shear (vertical) Example" /></td>
</tr>
<tr>
<td>Shear (horizontal)</td>
<td>$\begin{bmatrix} 1 &amp; s_h &amp; 0 \ 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \end{bmatrix}$</td>
<td>$x = v$ \quad $y = s_h v + w$</td>
<td><img src="image" alt="Shear (horizontal) Example" /></td>
</tr>
</tbody>
</table>
Image Rotation

FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.
Image registration

**FIGURE 2.37**
Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.
HW 1 -- Due: 9/26/2017

- Discussions are welcomed
- Never copy others’ work (no grades for both students)
Homework Requirements

- A word file with:
  - Questions
  - your solutions with output images
- M files (matlab) -- executable
- Email the word file and matlab files to our TA:
  - Mr. Xuejian Rong at
  - rxjjason@gmail.com
  - Subject: I2200 HW
Homework - presentation

The student will give about 15 mins to present the HW, then followed by 10 mins discussion

- HW1 – Adrian Logan
- HW2 – Mateusz Malinowski
- HW3 – Keshan Kissoon

A good opportunity to practice presentation skills + 5 extra bonus points for that HW!